個 體 經 濟 學 一

Microeconomics (I)

CH2 The analysis of consumer behavior

＊Untility function

Total utility of consuming (x, y), denoted as $u(x, y)$, is the total level of total satisfaction of consuming(x, y).

$$\begin{cases} \text{Cardinal utility analysis (number itself is meaningful)} \\ \text{Ordinal utility analysis (number is used to rank bundles)} \end{cases}$$

⇨ In Ordinal utility analysis, higher value of TU is associated with higher level of satisfaction.

utility function is a function from bundles to a real number

such that $\begin{cases} u(x1, y1) > u(x2, y2) \Leftrightarrow (x1, y1) > (x2, y2) \\ u(x1, y1) = u(x2, y2) \Leftrightarrow (x1, y1) \sim (x2, y2) \end{cases}$

$u : (x, y) \rightarrow \mathbb{R}$

EX: (2, 3) — 100

(3, 7) — 130

(5, 10) — 200

$u(x, y)$ total utility

＊Marginal utility (of X, of Y)

$u = u(x, y)$

$MU_x = \frac{\Delta u(x,y)}{\Delta x}$ $\left(\frac{\partial u(x,y)}{\partial x}\right)$

$MU_y = \frac{\Delta u(x,y)}{\Delta y}$ $\left(\frac{\partial u(x,y)}{\partial y}\right)$

⇨ The law of Diminishing marginal utility

$x\uparrow \quad MU_X\downarrow \ \left(\frac{\partial^2 u(x,y)}{\partial x^2} < 0 \right)$

$y\uparrow \quad MU_Y\downarrow \ \left(\frac{\partial^2 u(x,y)}{\partial y^2} < 0 \right)$

Any positively monotonic transformation of a utility function is also a utility function representing the same preference.

∗Cardinal v.s. Ordinal utility analysis

EX: x : # of toast

y: # of ham

$$u(x, y) = \min\left\{\frac{x}{3}, \frac{y}{2}\right\} \xrightarrow[\text{transformation}]{\text{positively monotonic}} \min\left\{\frac{x}{3}, \frac{y}{2}\right\}^2 + 100$$

=> same preference.

∗Marginal utility

$MU_x = \frac{\Delta u(x,y)}{\Delta x}$ $\quad$ $MU_y = \frac{\Delta u(x,y)}{\Delta y}$

or $\frac{\partial u(x,y)}{\partial x}$ $\quad$ or $\frac{\partial u(x,y)}{\partial y}$

or $U_x(x, y)$ $\quad$ or $U_y(x, y)$

(or simple U_x) $\quad$ (or simple U_y)

∗Marginal utility is the slope of the utility function

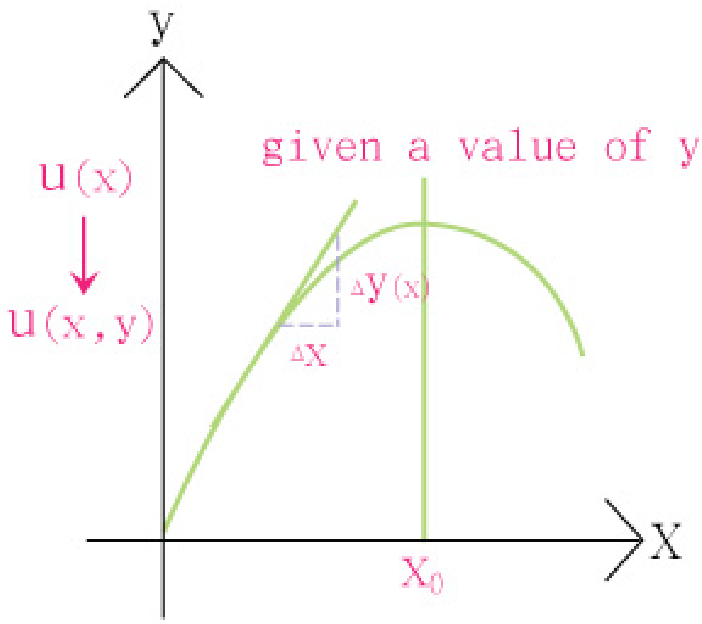
If more is better (assumption of nonsatiation)

$MU_x > 0$, $\quad MU_y > 0$

But "Law of Diminishing marginal utility"

$x\uparrow$, $MU_x \downarrow$

$y\uparrow$, $MU_y \downarrow$

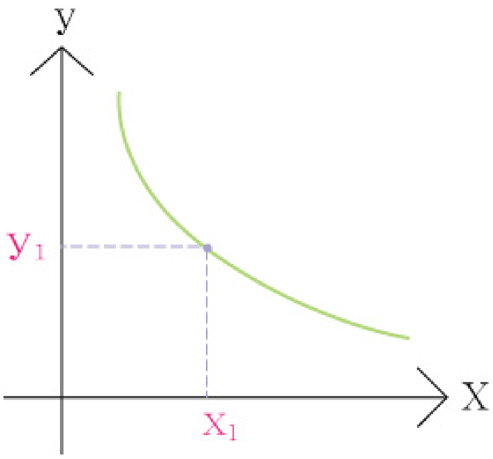


F igure1 8:

$$\{(x, y) | (x, y) \sim (x_1, y_1) \text{for all } (x, y) \in S\}$$

Given (x1, y1) ∈ S, we have total utility

$$u(x1, y1): \{(x, y) | u(x, y) = u(x_1, y_1) \text{for all } (x, y) \in S\}$$



F igure19 : Indifference curve

$MRS_{xy} = -\frac{\Delta y}{\Delta x}$ (given an indifference curve)

MRS_{xy} depends on (x, y)

note that $\Delta u(x, y) = \frac{\Delta u(x,y)}{\Delta x} \cdot \Delta x + \frac{\Delta u(x,y)}{\Delta y} \cdot \Delta y$

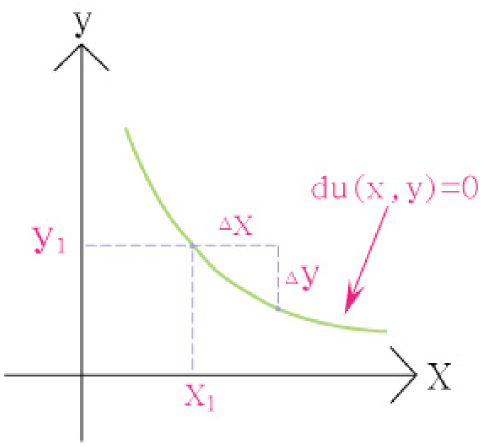
$$(du(x, y) = \frac{\partial u(x,y)}{\partial x} \cdot dx + \frac{\partial u(x,y)}{\partial y} \cdot dy)$$

$$du(x, y) = Mu_x \cdot dx + Mu_y \cdot dy$$

On an indifference curve, a change in X and Y must satisfy

$$Mu_x \cdot dx + Mu_y \cdot dy = 0$$

$$\Rightarrow \quad -\frac{dy}{dx} = \frac{Mu_x}{Mu_y}$$



F igure20 :

(note: $MRS_{xy} = -\frac{\Delta y}{\Delta x}$ of an indifference curve)

$$MRS_{xy}(x, y) = \frac{Mu_x(x,y)}{Mu_y(x,y)}$$

$\xRightarrow{\text{suppose}}$ $\left.\begin{array}{l} Mu_x(x,y) \text{ is diminishing on } X \\ Mu_y(x,y) \text{ is diminishing on } y \end{array}\right\}$ Diminishing marginal utility.

Can we have diminishing MRS_{xy}?

($MRS_{xy}\downarrow$ *as* $x\uparrow$ *or* $y\downarrow$)

$x\uparrow$ => $Mu_x\downarrow$(Diminishing Mu_x)

$y\downarrow$=> $Mu_y\uparrow$(stays on the same IC)

Since $MRS_{xy}\downarrow = \frac{Mu_x\downarrow}{Mu_y\uparrow}\downarrow$ as $x\uparrow$

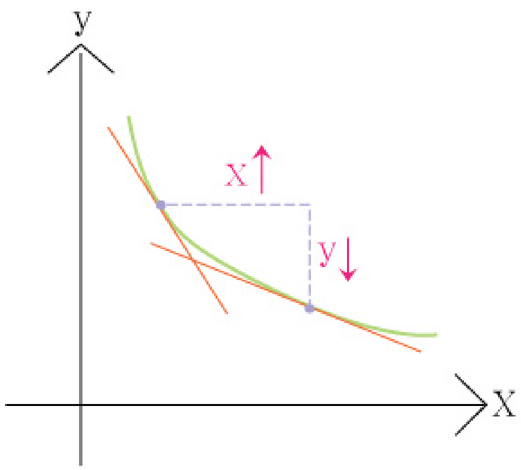


Figure21:Diminishing MRS_{xy}

$\frac{dMRS_{xy}}{dx} \gtrless 0$?

We hope $\frac{dMRS_{xy}}{dx} < 0$.

$Mu_x = u_x$ $Mu_y = u_y$ $\frac{\Delta Mu_x(x,y)}{\Delta x}$ or $\frac{\partial u_x}{\partial x} = u_{xx}$ $\frac{\Delta Mu_x(x,y)}{\Delta y}$ or $\frac{\partial u_x}{\partial y} = u_{xy}$ $\frac{\Delta Mu_y(x,y)}{\Delta x}$ or $\frac{\partial u_y}{\partial x} = u_{yx} = u_{xy}$ $\frac{\Delta Mu_y(x,y)}{\Delta y}$ or $\frac{\partial u_y}{\partial y} = u_{yy}$

$$\frac{d\mathrm{MRS_{xy}}}{dx} = \frac{d(\frac{\mathrm{Mu_x}}{\mathrm{Mu_y}})}{dx} = \frac{d(\frac{\mathrm{u_x}}{\mathrm{u_y}})}{dx} = \frac{\mathrm{u_y}\frac{du_x}{dx} - \mathrm{u_x}\frac{du_y}{dx}}{\mathrm{u_y}^2}$$

$$\left(\text{note that } \frac{\mathrm{du_x}}{\mathrm{dx}} \neq \frac{\partial \mathrm{u_x}}{\partial \mathrm{x}}\right) \quad = \frac{\mathrm{u_y}\left(\frac{\partial \mathrm{u_x}}{\partial \mathrm{x}}+\frac{\partial \mathrm{u_x}}{\partial \mathrm{y}}\cdot\frac{\mathrm{dy}}{\mathrm{dx}}\right)-\mathrm{u_x}\left(\frac{\partial \mathrm{u_y}}{\partial \mathrm{y}}+\frac{\partial \mathrm{u_x}}{\partial \mathrm{y}}\cdot\frac{\mathrm{dy}}{\mathrm{dx}}\right)}{\mathrm{u_y}^2}$$

a⟶c

$\frac{\partial \mathrm{u_x}}{\partial \mathrm{x}}$(given y)

c⟶b $\frac{\partial \mathrm{u_x}}{\partial \mathrm{x}}$ and $\frac{\mathrm{dx}}{\mathrm{dy}}$

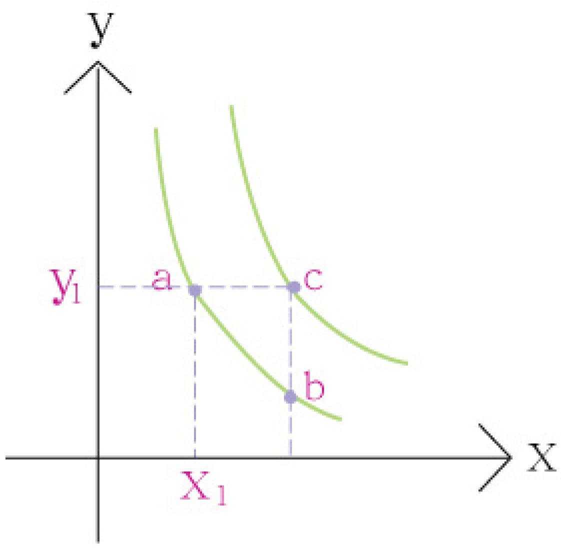


Figure22 :Diminishing $\mathrm{MRS_{xy}}$

(note that $-\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{u_x}}{\mathrm{u_y}}$ (= $\frac{\mathrm{Mu_x}}{\mathrm{Mu_y}}$))

⇨ $\frac{\mathrm{u_y}\left(\mathrm{u_{xx}}-\mathrm{u_{xy}}\cdot\frac{\mathrm{u_x}}{\mathrm{u_y}}\right)-\mathrm{u_x}\left(\mathrm{u_{yx}}-\mathrm{u_{yy}}\cdot\frac{\mathrm{u_x}}{\mathrm{u_y}}\right)}{\mathrm{u_y}^2}$

⇨ $\frac{\mathrm{u_y}^2\left(\mathrm{u_{xx}}-\mathrm{u_{xy}}\cdot\frac{\mathrm{u_x}}{\mathrm{u_y}}\right)-\mathrm{u_x u_y}\left(\mathrm{u_{yx}}-\mathrm{u_{yy}}\cdot\frac{\mathrm{u_x}}{\mathrm{u_y}}\right)}{\mathrm{u_y}^3}$

⇨ $\frac{\mathrm{u_y}^2\mathrm{u_{xx}}-2\mathrm{u_x u_y u_{xy}} +\mathrm{u_x}^2\mathrm{u_{yy}}}{\mathrm{u_y}^3} < 0$

none satisfaction $\mathrm{u_x}$, $\mathrm{u_y} > 0$

$\left.\begin{matrix}\mathrm{u_{xx}}<0\\ \mathrm{u_{yy}}<0\end{matrix}\right\}$ Diminishing $\mathrm{Mu_x}, \mathrm{Mu_y}$

Diminishing marginal utility is not sufficient to imply diminishing MRS_{xy}, we need to know $u_{xy} \begin{matrix}>0\\<0\end{matrix}$

∗Budget Constraint

X, Y　Quantity:x, y　　price: P_x, P_y　　income: m

Expenditure of$(x, y) = P_x \cdot x + P_y \cdot y$

Budget constraint: $\{(x,y) | P_x \cdot x + P_y \cdot y \leq m\}$

Budget line: $\{(x,y) | P_x \cdot x + P_y \cdot y = m\}$

$$P_x \cdot x + P_y \cdot y = m$$

$$P_y \cdot y = m - P_x \cdot x$$

$$y = \frac{m}{P_y} - \frac{P_x \cdot x}{P_y}$$

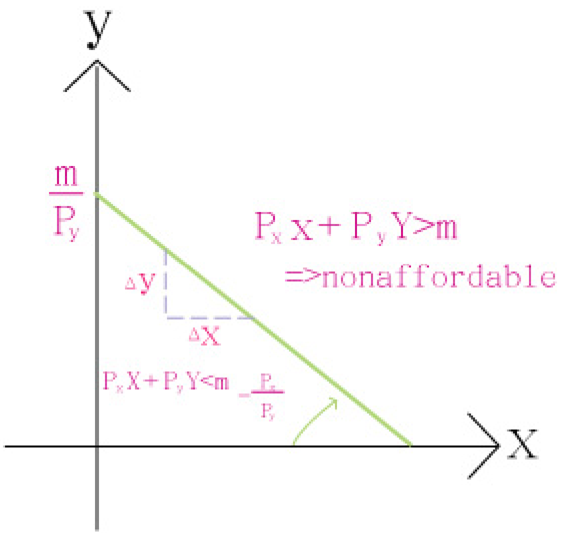


Figure23 :Budget constraint

$$MRSxy = -\frac{\Delta y}{\Delta x}\Big|_{\text{on an indifference curve}} = \text{subject exchange rate.}$$

$$\frac{P_x}{P_y} = -\frac{\Delta y}{\Delta x}\Big|_{\text{on an budget line}} = \text{object exchange rate.}$$

1. 所得(m)改變, m↑

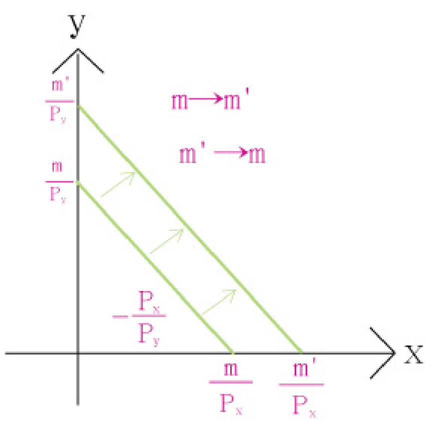


Figure24 :Budget constraint when income change

2. P_x, P_y chang

P_x↑, P_y fixed, m fixed

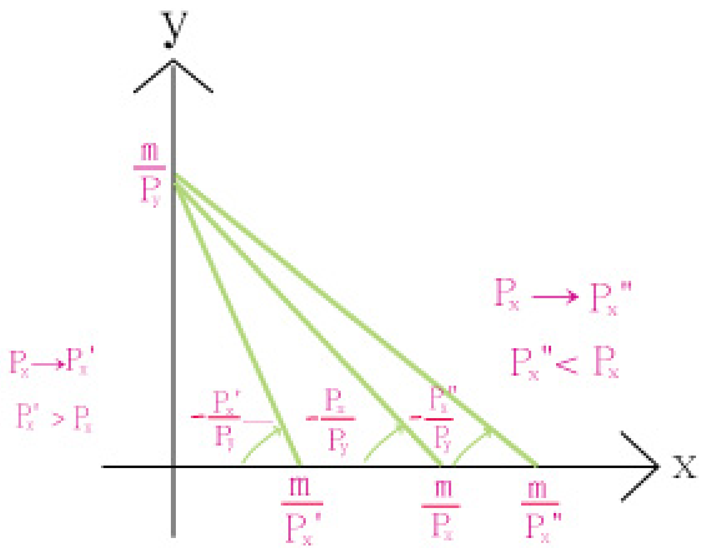


Figure25 :Budget constraint when price of X change(increase)

***Special case**

1.Quantity Discount on X

$$\begin{cases} X \leq X_0\ , P_x \\ X > X_0\ , \dfrac{P_x}{2} \end{cases}$$

m: income
Py:Price of gppd Y} as usual

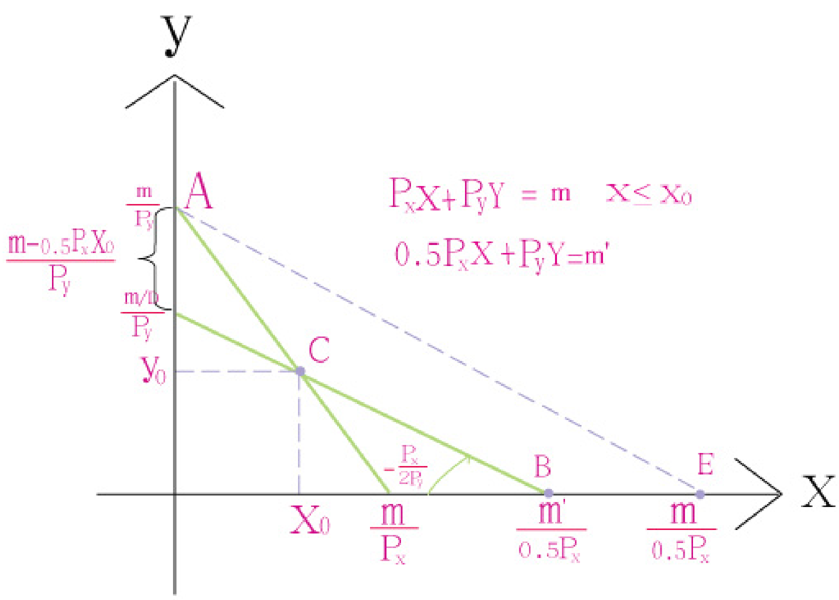


Figure26 :Budget constraint with special case

$$y_0 = \frac{m - P_x X_0}{P_y}$$

(x_0, y_0) is on D, B

$\left.\begin{array}{l}\text{Price of } X = 0.5\, P_x \\ \quad\quad\quad\quad = P_y\end{array}\right\}$ on D, B

income = m'

$$0.5 P_x X_0 + P_y \frac{m - P_x X_0}{P_y} = m'$$

$$0.5 P_x X_0 + m - P_x X_0 = m'$$

$$m - 0.5 P_x X_0 = m'$$

$$m - m' = 0.5 P_x X_0$$

- ⇨ Distance between A.D=$\frac{m - 0.5 P_x X_0}{P_y}$
- ⇨ D.B budget line = $\{(x, y) | 0.5 P_x X + P_y Y = m - 0.5 P_x X_0\}$
  $X > X_0$

2.Quota on X

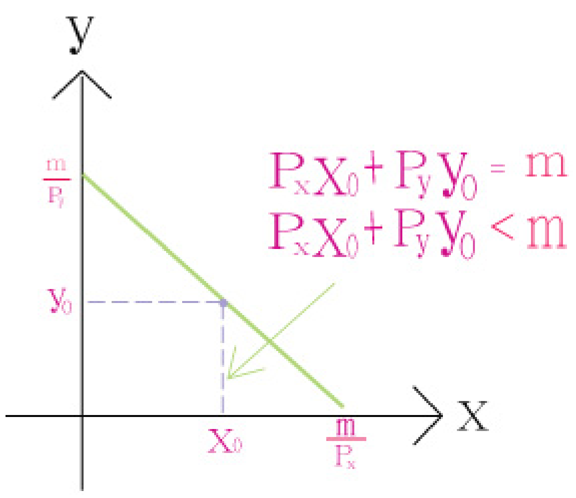
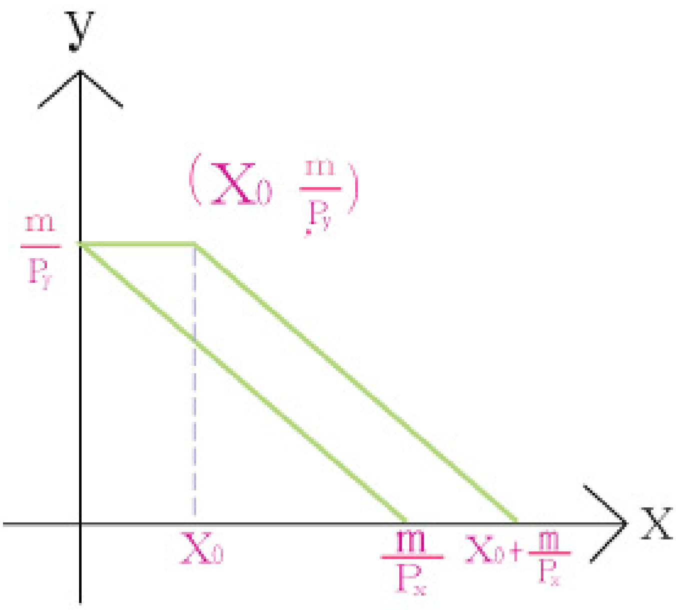


Figure27 :Budget constraint with quota

3. WIC(Woman.infant.children)
   Food coupon
   X_0:good X of food coupon



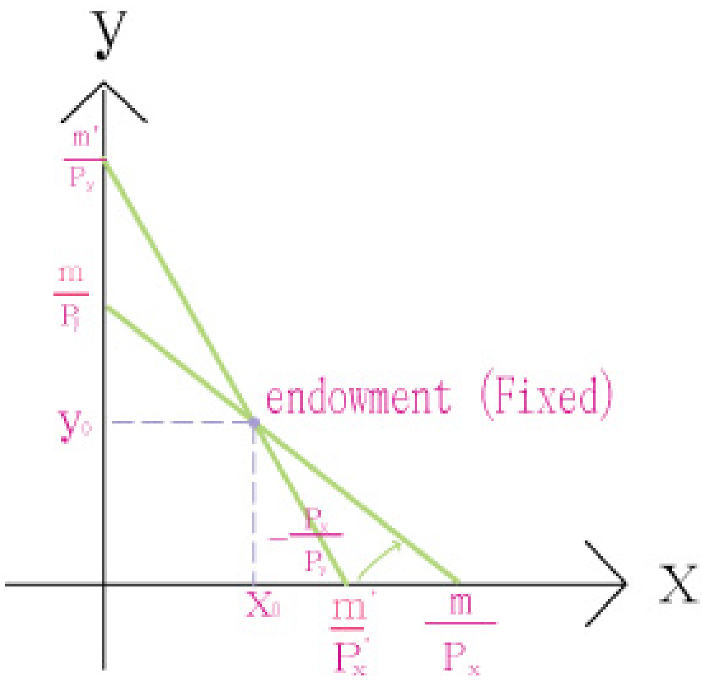
Ｆigure28 : Budget constraint with food coupon

4. Endowment 稟賦

   $(x0, y0) \sim m = P_x x_0 + P_y y_0$

   $P_x\uparrow$, $P_x \rightarrow P_x'$, $P_x' > P_x$

   $m' = P_x' X_0 + P_y Y_0$



Ｆigure29 :**Budget constraint with endowment**

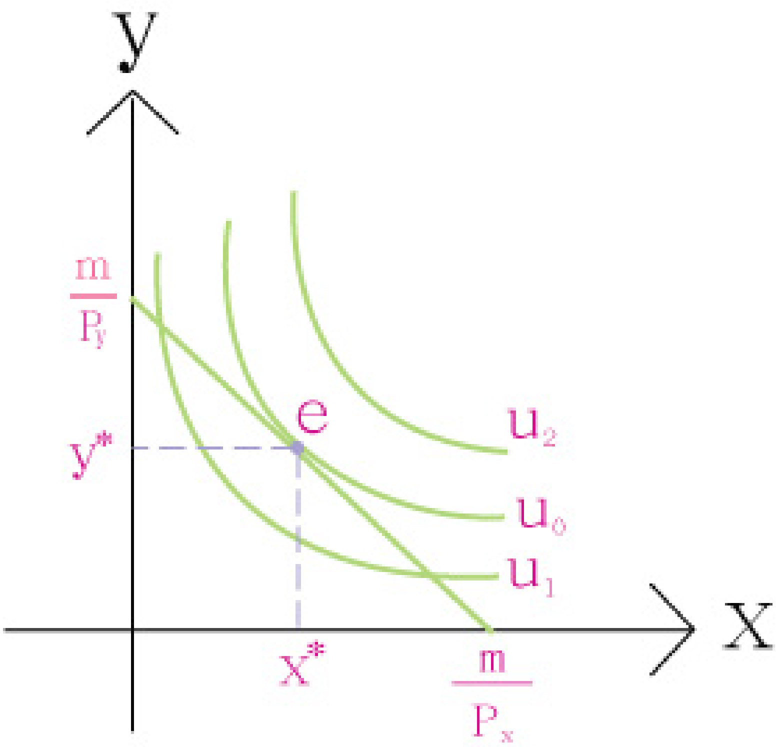


Figure30 :**Interior solution of consumer equilibrium**

∗Consumer equilibrium

A consumer maximizes his or her utility subjected to his/her budget constraint. *(X*, Y*)* is a consumer equilibrium where indifference curve is tangent to the budget line(indifference curve touches the budget line at e)
or slope of the indifference curve at e = slope of the budget line.

⇒$MRSxy(X^*, Y^*) = \frac{P_x}{P_y}$
or

◎one more condition: $P_x x^* + P_y y^* = m$

corner solution 角解

e: $x^*>0, y^*=0$

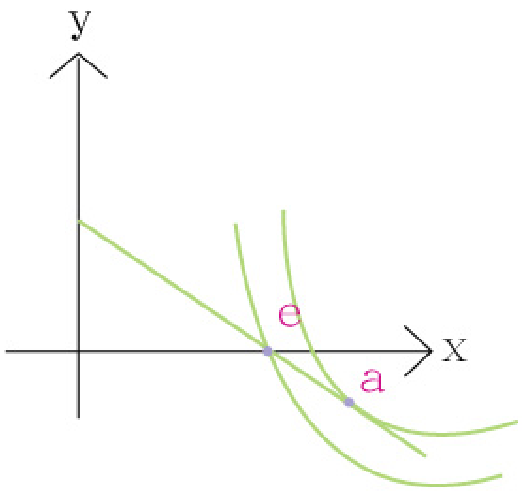
or $x^* = 0, \quad y^* = 0$

$P_x x^* + P_y y^* = m$

$P_x x^* = m, x^* = \frac{m}{P_x}$

$x^* = \frac{m}{P_x}$, $y^* = 0$, corner solution at e.

$MRS_{xy} \geq \frac{P_x}{P_y}$



Ｆigure31 :Corner solution of consumer equilibrium

another corner solution:

$x^{*}= 0, y* = \frac{m}{P_y}$, $MRS_{xy} \leq \frac{P_x}{P_y}$

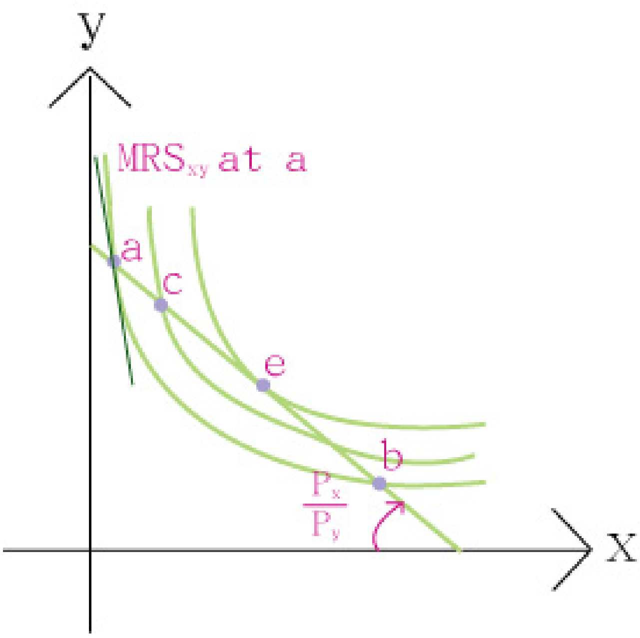
＊In the interior solution case:

if $MRS_{xy} > \frac{P_x}{P_y}$

subject change object exchange rate

(主觀)個人偏好 (客觀)價錢比

⇨ $x\uparrow, y\downarrow$



Ｆigure32 :

a→c

c has more X => c is better than a.

c→e

$MRS_{xy} = \frac{P_x}{P_y}$

同理, $MRS_{xy} < \frac{P_x}{P_y}$ => x↓, y↑

∗WIC case

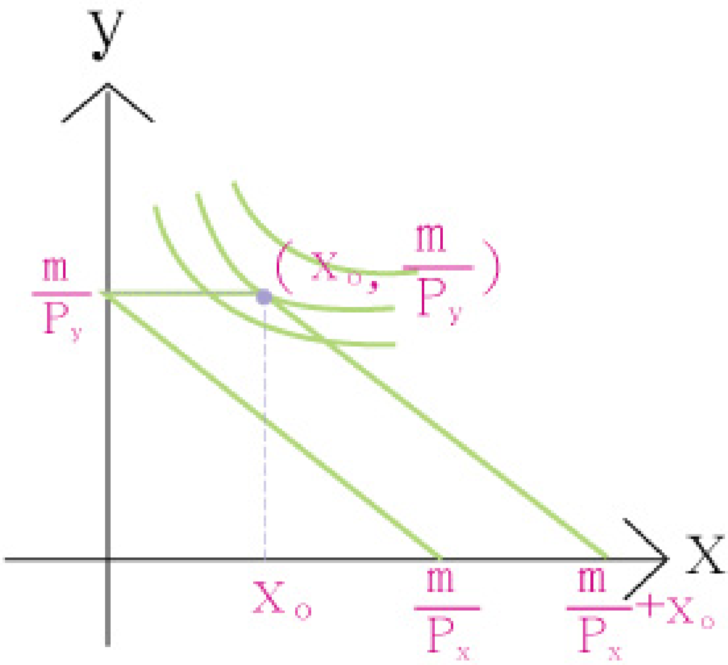


Figure33 :Consumer equilibrium with food coupon's type budget constraint

∗X and Y are perfect complements

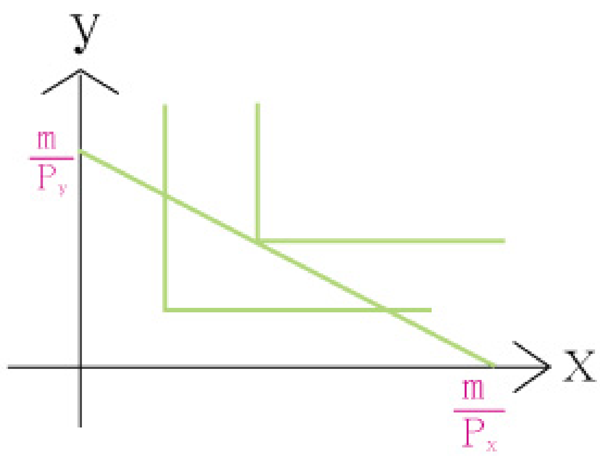


Figure34 :Interior solution under the preference is perfect complements

∗To max *u(x, y)* s.t $P_x \cdot x + P_y \cdot y = m$

u(x, y) is some continuously differentiable function

How to solve the consumer's problem?

⇨ General problem: max *f(x. y) s.t. g(x, y) = 0*

(min)

$$\mathcal{L}(x, y, \lambda) = u(x, y) + \lambda(m - P_x X - P_y Y) \quad \text{…Lagrange multiplier method}$$

$$= f(x, y) + \lambda g(x, y)$$

constrained problem →non-constrained problem and apply FOC to the non- constrained problem.

① $\frac{\partial \mathcal{L}(x,y,\lambda)}{\partial x} = \frac{\partial\left(u(x,y)+\lambda(m-P_x X-P_y Y)\right)}{\partial x} = \frac{\partial u(x,y)}{\partial x} - \lambda P_x = 0$

⇨ $\frac{\partial u(x,y)}{\partial x} = \lambda P_x$ --①'

② $\frac{\partial \mathcal{L}(x,y,\lambda)}{\partial y} = \frac{\partial\left(u(x,y)+\lambda(m-P_x X-P_y Y)\right)}{\partial y} = \frac{\partial u(x,y)}{\partial y} - \lambda P_y = 0$

⇨ $\frac{\partial u(x,y)}{\partial y} = \lambda P_x$ --②'

③ $\frac{\partial \mathcal{L}(x,y,\lambda)}{\partial \lambda} = \frac{\partial\left(u(x,y)+\lambda(m-P_x X-P_y Y)\right)}{\partial \lambda} = m - P_x x - P_y y = 0$ --③'

budget constraint

①'/②'= $\frac{Mu_x}{Mu_y} = \frac{P_x}{P_y}$ --④

$MRS_{xy} = \frac{Mu_x}{Mu_y}$

④ $MRS_{xy} = \frac{Mu_x}{Mu_y} = \frac{P_x}{P_y}$

在同樣的$1 上比較

$$\frac{Mu_x}{P_x} = \frac{Mu_y}{P_y} \Rightarrow \frac{Mu_x}{P_x} = \frac{\frac{\Delta u(x,y)}{\cancel{\Delta x}}}{\frac{\Delta expenditure}{\cancel{\Delta x}}}$$

∗ Special case

1. X and Y are perfect complement. 3 units of X is always consumed with 2 units of Y in fixed proportion. P_x=\$2, P_y=\$5, m=\$80.

$$u(x, y) = min\left\{\frac{x}{3}, \frac{y}{2}\right\}$$

Optimal combination of X&Y: $\frac{x}{3} = \frac{y}{2} \Rightarrow \frac{y}{x} = \frac{2}{3}$

$$2x + \frac{10}{3}x = 80$$

$$\frac{16}{3}x = 80$$

$$x^* = 15, y^* = 10$$

Generalized to $\min\left\{\frac{x}{a}, \frac{y}{b}\right\}$ →最不浪費的比例

s.t. $P_x x + P_y y = m$

Ex. X and Y are perfect complements. A consumer always uses 3 units of X with 2 units of Y in the fixed proportion.

$$\mathrm{u}(x, y) = \min\left\{\frac{x}{3}, \frac{y}{2}\right\} \Rightarrow \mathrm{u}(x, y) = \min\left\{\frac{x}{\mathrm{a}}, \frac{y}{\mathrm{b}}\right\}$$

$$\max \mathrm{u}(x, y) = \min\left\{\frac{x}{3}, \frac{y}{2}\right\}$$

s.t. $P_x x + P_y y = m$

$$\frac{x}{\mathrm{a}} = \frac{y}{\mathrm{b}} \Rightarrow y = \frac{\mathrm{b}}{\mathrm{a}}x$$

budget constraint: $P_x x + P_y y = m$

$$P_x x + P_y \frac{\mathrm{b}}{\mathrm{a}} x = m$$

$$\left(P_x + P_y \frac{\mathrm{b}}{\mathrm{a}}\right) x = m$$

$$x = \frac{m}{P_x + P_y \frac{\mathrm{b}}{\mathrm{a}}}$$

P_x=\$2, P_y=\$5

a=3, b=2, m=80

$$x^* = \frac{\mathrm{a}m}{\mathrm{a}P_x + \mathrm{b}P_y} = 15$$

$$y^* = \frac{\mathrm{b}}{\mathrm{a}} \frac{\mathrm{a}m}{\mathrm{a}P_x + \mathrm{b}P_y} = \frac{\mathrm{b}m}{\mathrm{a}P_x + \mathrm{b}P_y} = 10$$

Ex. Cobb-Douglas utility function

$u(x,y)_{\text{total utility}} = x^{\alpha}y^{\beta}, \alpha, \beta > 0$

$Mu_x = \frac{\partial u(x,y)}{\partial x} = \frac{\partial x^{\alpha}y^{\beta}}{\partial x} = \alpha x^{\alpha-1}y^{\beta} > 0$ no satiation point (no bliss point)

$\mathrm{Mu_y} = \frac{\partial \mathrm{u(x,y)}}{\partial \mathrm{y}} = \frac{\partial \mathrm{x}^{\alpha}\mathrm{y}^{\beta}}{\partial \mathrm{y}} = \beta \mathrm{x}^{\alpha}\mathrm{y}^{\beta-1} > 0$

$\frac{\partial Mu_x}{\partial x} = \alpha(\alpha-1)x^{\alpha-2}y^{\beta} \Rightarrow \alpha < 1$

$\frac{\partial Mu_y}{\partial y} = \beta(\beta-1)x^{\alpha}y^{\beta-2} \Rightarrow \beta < 1 \rightarrow Diminishing\ Marginal\ utility$

$MRS_{xy} = \frac{Mu_x}{Mu_y} = \frac{\alpha x^{\alpha-1}y^{\beta}}{\beta x^{\alpha}y^{\beta-1}} = \frac{\alpha}{\beta}\frac{y}{x} \rightarrow (x\text{越大}, y\text{越小}) \Rightarrow \frac{dMRS_{xy}}{dx} < 0$

For any α, β>0 and for all *x* and *y*

$\frac{dMRS_{xy}}{dx} < 0$

i.e. MRS_{xy} is diminishing (with *x*)

i.e. indifference curve is always convex

In equilibrium, $MRS_{xy} = \frac{P_x}{P_y}$

$\frac{\alpha}{\beta}\frac{y}{x} = \frac{P_x}{P_y}$

$P_y y(\text{expenditure on Y}) = \frac{\beta}{\alpha}P_x x(\text{expenditure on } X)$

$P_x x + P_y y = m$

$P_x x + \frac{\beta}{\alpha}P_x x = m$

$\frac{\alpha+\beta}{\alpha}P_x x = m \Longrightarrow e_x(\text{expenditure of x}) = P_x x = \frac{\alpha}{\alpha+\beta}m(\text{short of } e_x)$

$e_y(\text{expenditure of y}) = P_y y = \frac{\beta}{\alpha+\beta}m(\text{short of } e_y)$

$x^* = \frac{\alpha}{\alpha+\beta}\frac{m}{P_x}$

$y^* = \frac{\beta}{\alpha+\beta}\frac{m}{P_y}$